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A Hybrid FEM-BEM Unified Boundary Condition with Sub-Cycling for Electromagnetic Radiation

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Introduction

Hybrid solutions to time-domain electromagnetic problems offer many advantages when solving open-region scattering or radiation problems. Hybrid formulations use a finite-element or finite-difference discretization for the features of interest, then bound this region with a layer of planar boundary elements. The use of volume discretization allows for intricate features and many changes in material within the structure, while the boundary-elements provide a highly accurate radiating boundary condition. This concept has been implemented previously, using the boundary elements to set the E-field, H-field, or both for an FDTD grid, for example in [1][2][3], or as a mixed boundary condition for the second order wave equation solved by finite elements [4]. Further study has focused on using fast methods, such as the Plane Wave Time Domain method [3][4] to accelerate the BEM calculations.

This paper details a hybrid solver using the coupled first-order equations for the E and H fields in the finite-element region. This formulation is explicit, with a restriction on the time step for stability. When this time step is used in conjunction with the boundary elements forming either a inhomogenous Dirichlet or Neuman boundary condition on the finite-element mesh, late time instabilities occur. To combat this, a Unified Boundary Condition (UBC), similar to the one in [4] for the second-order wave equation, is used. Even when this UBC is used, the late time instabilities are merely delayed if standard testing in time is used. However, the late time instabilities can be removed by replacing centroid based time interpolation with qaudrature point based time interpolation for the boundary elements, or by sub-cycling the boundary element portion of the formulation. This sub-cycling, used in [3] for FDTD to reduce complexity, is shown here to improve stability and overall accuracy of the technique.

Formulation

The finite-element portion of the code uses a hexahedron mesh. The electric field is represented on the mesh using discrete differential 1-form edge basis functions, while the magnetic flux is represented using discrete differential 2-form face basis functions[5]. At each time step, Ampere's law

$$\frac{\partial(\varepsilon \boldsymbol{E})}{\partial t} = \nabla \times \mu^{-1} \boldsymbol{B} - \boldsymbol{J} \tag{1}$$

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with **E** the electric field, **B** the magnetic flux density, and **J** any impressed source currents, is solved using central-difference discretization in time. The magnetic flux density is then updated from

$$\frac{\partial(\boldsymbol{B})}{\partial t} = -\nabla\times\boldsymbol{E} - \boldsymbol{J} \tag{2}$$

without the need for a matrix solve. When (1) is discretized, tested with the testing function **w,** and integration-by-parts is applied, the equation to be solved becomes

$$\int_V \frac{\partial(\varepsilon\boldsymbol{E})}{\partial t}\bullet\boldsymbol{w} = -\int_S \mu^{-1}\boldsymbol{w}\bullet(\boldsymbol{B}\times\hat{\boldsymbol{n}}) + \int_V \mu^{-1}\boldsymbol{B}\bullet\nabla\times\boldsymbol{w} - \int_V \boldsymbol{J}\bullet\boldsymbol{w}\cdot \tag{3}$$

To accurately evaluate radiating boundary conditions, a two-surface boundary element formulation is used. The equivalent magnetic and electric currents, **J** and **M** are computed on an inner surface slightly inside the outer boundary of the mesh. These currents are represented as

$$\mathbf{J}(\mathbf{r},t) = \sum_{n=1}^{N}\sum_{j=1}^{T} I_n^{(j)}\ \mathbf{\Lambda}_n(\mathbf{r})T_j(t) \tag{4}$$

where N is the number of surface elements on the inner surface, $T_j(t)$ is a linear time basis function, $\mathbf{\Lambda}_n(\mathbf{r})$ is a divergence-conforming surface basis function, and $I_n^{(j)}$ is a coefficient for the nth spatial basis function at time step j found from the finite-element **E** and **H** fields. These equivalent currents are used to find either the electric or magnetic field by first representing the fields in terms of potentials,

$$\boldsymbol{E} = -\frac{\partial\boldsymbol{A}}{\partial t} - \nabla\Phi - \frac{1}{\varepsilon}\nabla\times\boldsymbol{F} \qquad \boldsymbol{H} = -\frac{\partial\boldsymbol{F}}{\partial t} - \nabla\Psi + \frac{1}{\mu}\nabla\times\boldsymbol{A}, \tag{5}$$

then expressing the potentials in terms of the equivalent currents as

$$\frac{\partial\mathbf{A}(\mathbf{r},t)}{\partial t} = \frac{\mu}{4\pi}\int_S \frac{\dot{\mathbf{J}}(\mathbf{r}',\tau)}{R}dS' \qquad \frac{\partial\boldsymbol{F}(\mathbf{r},t)}{\partial t} = \frac{\varepsilon}{4\pi}\int_S \frac{\dot{\boldsymbol{M}}(\mathbf{r}',\tau)}{R}dS' \tag{6}$$

$$\Phi(\mathbf{r},t) = -\frac{1}{4\pi\varepsilon}\int_S\int_0^\tau \frac{\nabla_S'\cdot\mathbf{J}(\mathbf{r}',t')}{R}dt'dS' \qquad \Psi(\mathbf{r},t) = -\frac{1}{4\pi\mu}\int_S\int_0^\tau \frac{\nabla_S'\cdot\boldsymbol{M}(\mathbf{r}',t')}{R}dt'dS' \quad .$$

The fields at the outer boundary can be applied used to create several types of boundary conditions for the finite element mesh. The electric field can be used for Dirichlet boundary conditions on **E**, $-\hat{\boldsymbol{n}}\times\hat{\boldsymbol{n}}\times\boldsymbol{E}|_S = \boldsymbol{E}(\boldsymbol{J},\boldsymbol{M})$. The magnetic field can be used to impose a Neumann boundary condition by $-\int_S \mu^{-1}\mathbf{w}\bullet(\boldsymbol{B}\times\hat{\boldsymbol{n}}) \rightarrow -\int_S \hat{\boldsymbol{n}}\times\boldsymbol{w}\bullet\boldsymbol{H}(\boldsymbol{J},\boldsymbol{M})\cdot$

Alternately, both can be used for a unified boundary condition given by $c\hat{\boldsymbol{n}}\times\hat{\boldsymbol{n}}\times\boldsymbol{E} + \hat{\boldsymbol{n}}\times\boldsymbol{B} = \boldsymbol{U}$, where c is the speed of light and the integration-by-parts term is replaced with $-\int_S \mu^{-1}\mathbf{w}\bullet(\boldsymbol{B}\times\hat{\boldsymbol{n}}) = -\frac{1}{Z}\int_S(\hat{\boldsymbol{n}}\times\boldsymbol{w})\bullet(\hat{\boldsymbol{n}}\times\boldsymbol{E}) + \int_S \mu^{-1}\mathbf{w}\bullet(\boldsymbol{U})$

where $\boldsymbol{U} = -\frac{\boldsymbol{1}}{c}\hat{\boldsymbol{n}}\times\hat{\boldsymbol{n}}\times\boldsymbol{E}(\boldsymbol{J},\boldsymbol{M}) + \hat{\boldsymbol{n}}\times\boldsymbol{B}(\boldsymbol{J},\boldsymbol{M})\cdot$

The time step for the integral equation (the width of the temporal basis functions $T_j(t)$) need not be the same time step as the finite element time step. In particular, it can be sub-cycled at some larger multiple S of the finite element time step. This offers two advantages. The first is that this sub-sampling leads to faster time-stepping, because the matrix-vector products used to compute E and H on the outer boundary do not need to be performed every time step. The second advantage is that the representation of E and H on the boundary can actually become more accurate, leading to a more stable solution. Because the time step needed for the finite element is often small, such that light takes several timesteps to cross one face of an element, if standard centroid testing in time is used for the time basis functions, the resulting integration can be fairly inaccurate. In contrast, when centroid testing in time is used in pure BEM codes, the time step must be chosen significantly larger than an element. By using a multiple of the finite element time step for the BEM timestep, the centroid testing in time becomes more accurate.

Another solution to this problem is to sample the time basis function by quadrature point, rather than by patch centroids. This does improve the accuracy, but has the unfortunate affect of increasing the number of non-zeros in the potential matrices, greatly increasing the time needed for each timestep.

Results

The test case considered is that of radiation from an infinitesimal dipole. A 2m x 2m Cartesian mesh centered at the origin was formed with 16 cubic finite elements along each dimension. The center 4x4x4 block of elements was removed, and the exact solution for the near fields of a ramped-on dipole were impressed on this surface as a Dirichlet boundary condition. The inner surface used for computing equivalent currents was located one cell inside the outer boundary. A normalized unit system was used, with permittivity and permeability both set to unity. A timestep of 2.5e-2 seconds was used. The electric field was sampled within the finite element region at the location (0.4375, 0.5625, 0.5625). The problem set-up is depicted in Figure 1.

The problem was simulated using several different formulations. The E-field hybrid boundary, H-field hybrid boundary, the UBC, and standard first-order ABC were all tested. In addition, for the UBC, the problem was run with a sub-cycling factor on the boundary element components of 2 and 4 times the finite element timestep. The results for number time to fill the BEM matrix, time per timestep, average accuracy in the magnitude of E, and number of timesteps until instability are all shown in Table 1. All times shown are in seconds, using 16 processors of a Linux-based supercomputer. It can be seen from the table that filling by centroid in time is significantly faster than filling by quadrature point, and can still produce a stable solution if sub-cycling on the boundary elements is performed.

Table 1. Results for an infintesimal dipole

Formulation	BEM Fill Time	Time / Timestep	Average % Error	Timesteps of Stability
ABC	----	2.40E-02	10.22	12000+
E-Field Hybrid	210	5.98E-01	2.88	1984
H-Field Hybrid	212	3.40E-01	3.73	1040
UBC, Quadrature-In-Time	214	5.95E-01	1.73	12000+
UBC, Centroid-In-Time	38	2.55E-01	1.77	8336
UBC, Centroid-In-Time, S=2	37	1.31E-01	1.54	12000+
UBC, Centroid-In-Time, S=4	36	7.30E-02	1.51	12000+

Conclusion

A hybrid Unified Boundary Condition for FEM/BEM analysis using coupled first order equations was presented and results given. The results show that that a UBC provides stability, which is not found in formulations which only satisfy the E or H field on the radiating boundary. In addition, speed improvements were obtained by using centroid-in-time testing and sub-cycling the BEM portion of the simulation.

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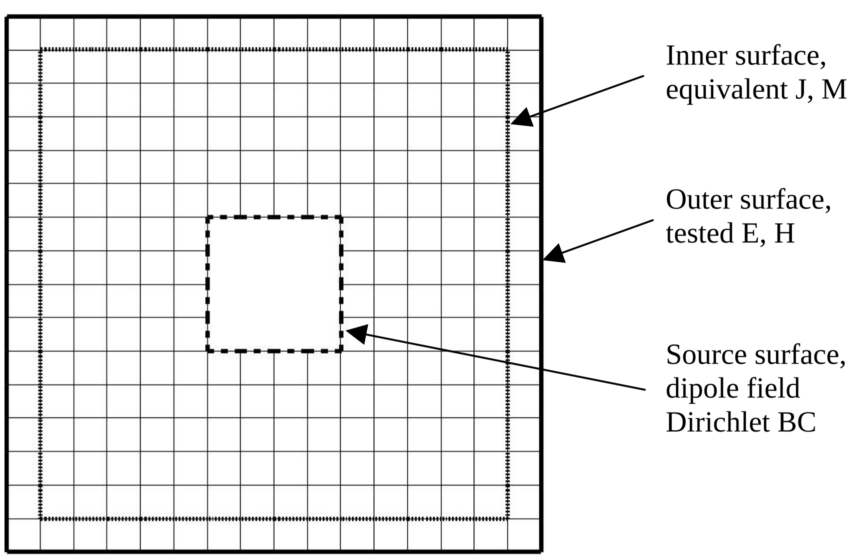

Figure 1. Cross-section of the mesh used for simulating radiation from an infinitesimal dipole.